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Group: _____

Lecturer: A.S. Eremenko

HOMEWORK 9

1. The conditional PMF of a random variable X , conditioned on a particular event A with $P(A) > 0$, is defined by:

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a) $p_X(k) = P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$;

b) $p_X(x) = P(\{X = x\})$;

c) $p_{X|A}(x) = P(X = x|A) = \frac{P(\{X=x\} \cap A)}{P(A)}$;

d) $p_{X|A}(x) = P(\{X = x\})$.

2. The conditional PDF of a continuous random variable X , conditioned on a particular event A with $P(A) > 0$, is a function $f_{X|A}$ defined as:

a) $p_{X|A}(x) = P(X = x|A) = \frac{P(\{X=x\} \cap A)}{P(A)}$;

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b) $P(X \in B|A) = \int_B f_{X|A}(x) dx$;

c) $f_{X|A}(x) = P(\{X = x\})$;

d) $f_{X|A}(x|A) = \begin{cases} \frac{f_X(x)}{P(X \in A)} & \text{if } x \in A, \\ 0 & \text{otherwise.} \end{cases}$

3. This is the total expectation theorem for discrete and continuous random variables:

a) $E[X] = \sum_{i=1}^n P(A_i) E[X|A_i]$;

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b) $E[X] = \sum_{i=1}^n P(A_i) E[g(X)|A_i]$;

c) $E[X] = \sum_x g(x) p_X(x)$;

d) $E[X] = \int_{-\infty}^{\infty} g(x) f_X(x) dx$.

4. Variance of the discrete uniform random variable is defined as:

a) $\text{var}[X] = \frac{a+b}{2}$;

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b) $\text{var}(X) = \frac{(b-a)^2}{12}$;

c) $\text{var}[X] = \frac{(b-a)(b-a+2)}{12}$;

d) $\text{var}[X] = \frac{(b-a)(b-a+1)}{12}$.

5. The cumulative distribution function of exponential random variable is given by:

a) $F_X(x) = 1 - e^{-\lambda x}$ if $x \geq 0$;

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b) $F_X(x) = e^{-\lambda x}$ if $x \geq 0$;

c) $F_X(x) = \lambda e^{-\lambda x}$ if $x \geq 0$;

d) $F_X(x) = e^{-\lambda x}$ if $x \geq a$.

6. Normal or Gaussian random variable has a PDF of the form:

a) $f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$;

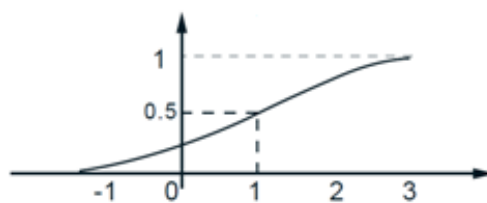
b) $f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} e^{-(x-\mu)^2/2\sigma^2} dx$;

c) $f_X(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^y e^{-\frac{t^2}{2}} dt$;

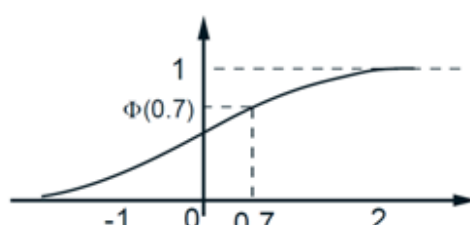
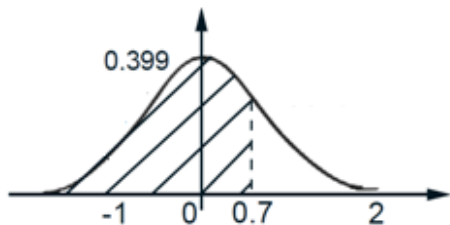
d) $f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$.

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7. Indicate x -axis, y -axis and functions of the following graphs:



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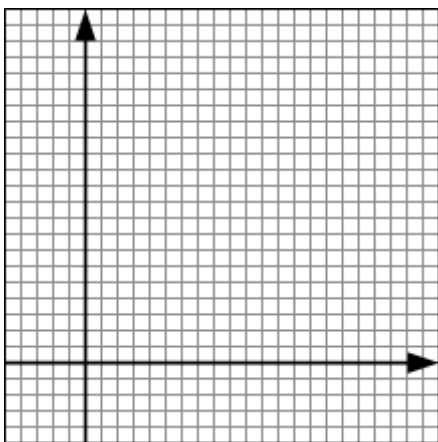
Problem 1. Let X be a random variable with PDF

$$f_X(x) = \begin{cases} x^2 & \text{if } 0 \leq x \leq 2, \\ 0 & \text{otherwise,} \end{cases}$$

and let A be the event $\{X \geq 1\}$. Find $E[X]$, $P(A)$, $f_{X|A}(x|A)$ and $E[X|A]$. Show graphically $f_X(x)$, $P(A)$ and $f_{X|A}(x|A)$.

Solution:

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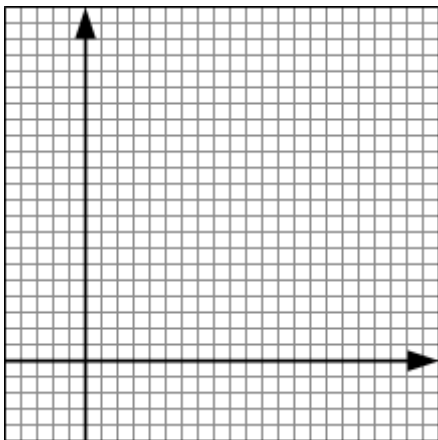


Problem 2. Operator of a call center during 30 min processes with equal probability from 20 to 25 customers calls. Let X be the RV of number of calls during that period. What

distribution has X ? Find its PMF. Let event A such that operator processed more than 23 calls during last 30 min period. Find conditional PMF $p_{X|A}(x)$. Show graphically all PMFs.

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Solution:



Problem 3. Find expectation, variance and standard deviation of exponential time between failures on the ATM switch with $\lambda = 5$ failures per hour.

Solution:

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Problem 4. Round-trip time (RTT) within the TCP-connection has a normal distribution with mean 250 ms and a standard deviation of $\sigma = 10$ ms. Find probabilities that $RTT \leq 270$, $RTT > 245$, $255 \leq RTT \leq 260$.

Solution:

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Probability Content from $-\infty$ to Z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990