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HOMEWORK 11

1. The joint PMF of X and Y is defined by:

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a) $p_{X,Y}(x,y) = P(X = x, Y = y)$;

b) $p_{X,Y}(x,y) = P(\{X = x\})$;

c) $p_{X,Y}(x,y) = P(\{X = x\} \cap \{Y = y\})$;

d) $p_{X,Y}(x,y) = P(X = x \; and \; Y = y)$.

2. How can be obtained marginal PMFs of X and Y from the joint PMF:

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a) $p_X(x) = \sum_{x,y} g(x,y) p_{X,Y}(x,y)$;

b) $p_Y(y) = \sum_{x,y} g(x,y) p_{X,Y}(x,y)$;

c) $p_X(x) = \sum_{y} p_{X,Y}(x,y)$;

d) $p_Y(y) = \sum_{x} p_{X,Y}(x,y)$?

3. For any random variables $X_1, X_2, \dots, X_n$ and any scalars $a_1, a_2, \dots, a_n$, we have the expected value rule:

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a) $E[X] = \sum_{i=1}^{n} P(A_i) E[X|A_i]$;

b) $E[a_1 X_1 + a_2 X_2 + \cdots + a_n X_n] = a_1 E[X_1] + a_2 E[X_2] + \cdots + a_n E[X_n]$;

c) $E[X] = \sum_x g(x) p_X(x)$;

d) $E[g(X,Y,Z)] = \sum_{x,y,z} g(x,y,z) p_{X,Y,Z}(x,y,z)$.

4. A joint probability density function $f_{X,Y}$ is a nonnegative function that satisfies:

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a) $p_{X,Y}(x,y) = P(\{X = x\} \cap \{Y = y\})$;

b) $P\big((X,Y) \in B\big) = \iint_{(X,Y)\in B} f_{X,Y}(x,y)\,dxdy$ for every subset B of the two-dimensional plane;

c) $f_{X,Y}(x,y) = \frac{\partial^2 F_{X,Y}}{\partial x \partial y}(x,y)$ where $F_{X,Y}$ joint CDF;

d) $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dxdy = 1$ if B is the entire two-dimensional plane.

5. Conditioning one random variable on another can be defined by:

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a) $f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$;

b) $f_{X|A}(x|A) = \begin{cases} \frac{f_X(x)}{P(X\in A)} \; if \; x \in A, \\ 0 \; otherwise. \end{cases}$;

c) $p_{X|Y}(x|y) = \frac{P(X=x, Y=y)}{P(Y=y)} = \frac{p_{X,Y}(x,y)}{p_Y(y)}$;

d) $p_{X|A}(x) = P(X = x|A) = \frac{P(\{X=x\}\cap A)}{P(A)}$.

6. Which from the listed below statements are true for independent random variables:

a) $p_{X,Y}(x,y) = p_X(x)p_Y(y)$, for all x, y; 1

b) $f_{X,Y}(x,y) = f_X(x)f_Y(y)$, for all x, y;

c) $E[XY] = E[X]E[Y]$;

d) $\mathrm{var}(X+Y) = \mathrm{var}(X) + \mathrm{var}(Y)$.

7. Find the correct formula of covariance and correlation coefficient of two random variables X and Y:

a) $\rho \ = \frac{\mathrm{cov}\,(X,Y)}{\sqrt{\mathrm{var}\,(X)\mathrm{var}\,(Y)}}$, X and Y have nonzero variances; 1

b) $\mathrm{cov}(X,Y) = 0$;

c) $\mathrm{cov}(X,Y) = E[XY] - E[X]E[Y]$;

d) $\mathrm{cov}(X,Y) = E[(X - E[X])(Y - E[Y])]$.

Problem 1. Delay of data transfer from the node of customer access network is a discrete random variable. Two random variables X and Y are delays from two nodes and have the following joint PMF:

Y_i / X_i	0	1	5
1	0.1	0.2	0
3	0	0.3	0
4	0.1	0.2	0.1

Find numerical characteristics: $E[X]$, $E[Y]$, $\mathrm{var}[X]$, $\mathrm{var}[Y]$, $\mathrm{cov}(X,Y)$ and $\rho(X,Y)$. Show graphically $P_X(x)$ and $P_Y(y)$. 8

Solution:

Problem 2. Two applications generate two flows of data. RV X characterizes time between data units departures of the first flow with $f_X(x) = 0.001 \cdot e^{-0.001t}$, and RV Y analogous time parameter for the second flow with $f_Y(y) = 0.005 \cdot e^{-0.005t}$. RVs X and Y are independent from each other. Find joint PDF $f_{X,Y}(x, y)$, $E[XY]$ and $\mathrm{var}(X + Y)$. Suppose that flow intensity dimension is $[byte\ per\ second]$.

Solution:

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