

Student: _____

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HOMEWORK 11

1. The joint PMF of X and Y is defined by:

a) $p_{X,Y}(x, y) = P(X = x, Y = y);$

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b) $p_{X,Y}(x, y) = P(\{X = x\});$

c) $p_{X,Y}(x, y) = P(\{X = x\} \cap \{Y = y\});$

d) $p_{X,Y}(x, y) = P(X = x \text{ and } Y = y).$

2. How can be obtained marginal PMFs of X and Y from the joint PMF:

a) $p_X(x) = \sum_{x,y} g(x, y)p_{X,Y}(x, y);$

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b) $p_Y(y) = \sum_{x,y} g(x, y)p_{X,Y}(x, y);$

c) $p_X(x) = \sum_y p_{X,Y}(x, y);$

d) $p_Y(y) = \sum_x p_{X,Y}(x, y)?$

3. For any random variables X_1, X_2, \dots, X_n and any scalars a_1, a_2, \dots, a_n , we have the expected value rule:

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a) $E[X] = \sum_{i=1}^n P(A_i)E[X|A_i];$

b) $E[a_1X_1 + a_2X_2 + \dots + a_nX_n] = a_1E[X_1] + a_2E[X_2] + \dots + a_nE[X_n];$

c) $E[X] = \sum_x g(x)p_X(x);$

d) $E[g(X, Y, Z)] = \sum_{x,y,z} g(x, y, z)p_{X,Y,Z}(x, y, z).$

4. A joint probability density function $f_{X,Y}$ is a nonnegative function that satisfies:

a) $p_{X,Y}(x, y) = P(\{X = x\} \cap \{Y = y\});$

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b) $P((X, Y) \in B) = \iint_{(X,Y) \in B} f_{X,Y}(x, y) dx dy$ for every subset B of the

two-dimensional plane;

c) $f_{X,Y}(x, y) = \frac{\partial^2 F_{X,Y}}{\partial x \partial y}(x, y)$ where $F_{X,Y}$ joint CDF;

d) $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,Y}(x, y) dx dy = 1$ if B is the entire two-dimensional plane.

5. Conditioning one random variable on another can be defined by:

a) $f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)};$

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b) $f_{X|A}(x|A) = \begin{cases} \frac{f_X(x)}{P(X \in A)} & \text{if } x \in A, \\ 0 & \text{otherwise.} \end{cases}$

c) $p_{X|Y}(x|y) = \frac{P(X=x, Y=y)}{P(Y=y)} = \frac{p_{X,Y}(x,y)}{p_Y(y)};$

d) $p_{X|A}(x) = P(X = x|A) = \frac{P(\{X=x\} \cap A)}{P(A)}.$

6. Which from the listed below statements are true for independent random variables:

a) $p_{X,Y}(x, y) = p_X(x)p_Y(y)$, for all x, y ;

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b) $f_{X,Y}(x, y) = f_X(x)f_Y(y)$, for all x, y ;

c) $E[XY] = E[X]E[Y]$;

d) $\text{var}(X + Y) = \text{var}(X) + \text{var}(Y)$.

7. Find the correct formula of covariance and correlation coefficient of two random variables X and Y :

a) $\rho = \frac{\text{cov}(X,Y)}{\sqrt{\text{var}(X)\text{var}(Y)}}$, X and Y have nonzero variances;

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b) $\text{cov}(X, Y) = 0$;

c) $\text{cov}(X, Y) = E[XY] - E[X]E[Y]$;

d) $\text{cov}(X, Y) = E[(X - E[X])(Y - E[Y])]$.

Problem 1. Delay of data transfer from the node of customer access network is a discrete random variable. Two random variables X and Y are delays from two nodes and have the following joint PMF:

$Y_i \backslash X_i$	0	1	5
1	0.1	0.2	0
3	0	0.3	0
4	0.1	0.2	0.1

Find numerical characteristics: $E[X]$, $E[Y]$, $\text{var}[X]$, $\text{var}[Y]$, $\text{cov}(X, Y)$ and $\rho(X, Y)$. Show graphically $P_X(x)$ and $P_Y(y)$.

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Solution:

Problem 2. Two applications generate two flows of data. RV X characterizes time between data units departures of the first flow with $f_X(x) = 0.001 \cdot e^{-0.001t}$, and RV Y analogous time parameter for the second flow with $f_Y(y) = 0.005 \cdot e^{-0.005t}$. RVs X and Y are independent from each other. Find joint PDF $f_{X,Y}(x, y)$, $E[XY]$ and $\text{var}(X + Y)$. Suppose that flow intensity dimension is [*byte per second*].

Solution:

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