Student: ________________________________

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HOMEWORK 10

1. If we have two independent normal random variables with zero mean and variances σ^2, what distribution has random variable $R = \sqrt{X^2 + Y^2}$:

1	

a) normal;

b) chi-square;

c) Rayleigh;

d) Maxwellian?

2. Gauss error function used to define the cumulative distribution function of:

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a) Rayleigh random variable;

b) lognormal random variable;

c) Maxwell-Boltzmann random variable;

d) chi-square random variable.

3. Which theoretical probability distribution is one of the most widely used in inferential statistics and in statistical significance tests:

1	

a) normal;

b) chi-square;

c) Rayleigh;

d) Maxwellian?

4. If X is discrete with PMF p_X and $Y = g(X)$ is a function of a random variable X, then Y is also discrete, and its PMF p_Y can be calculated using the PMF of X using:

1	

a) $p_Y(y) = \sum_x p_X(x)$;

b) adding the probabilities of all values of x such that $g(x) = y$;

c) $p_Y(y) = \sum_{\{x \mid g(x)=y\}} p_X(x)$;

d) $F_Y(y) = P(g(X) \leq y) = \int_{\{x \mid g(x)\leq y\}} f_X(x)dx$.

5. Calculation of the PDF of a function $Y = g(X)$ of a continuous random rariable X consists of two following steps:

1	

a) $F_Y(y) = P(g(X) \leq y) = \int_{\{x \mid g(x)\leq y\}} f_X(x)dx$;

b) adding the probabilities of all values of x such that $g(x) = y$;

c) $p_Y(y) = \sum_{\{x \mid g(x)=y\}} p_X(x)$;

d) $f_Y(y) = \frac{dF_Y}{dy}(y)$.

6. The PDF of a linear function $Y = aX + b$ (for some scalars $a \neq 0$ and b) of a random variable X is:

a) $f_Y(y) = \frac{dF_Y}{dy}(y)$;

1	

b) $F_Y(y) = P(Y \leq y) = P(aX + b \leq y) = P\left(X \leq \frac{y-b}{a}\right) = F_X\left(\frac{y-b}{a}\right)$;

c) $f_Y(y) = \frac{1}{|a|} f_X\left(\frac{y-b}{a}\right)$;

d) $f_Y(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2}$.

7. Rule of obtaining a PDF for a monotonic function of a continuous random variable consists of following statements:

1	

a) $F_Y(y) = P(g(X) \leq y) = \int_{\{x \mid g(x) \leq y\}} f_X(x)dx$;

b) $f_Y(y) = \frac{dF_Y}{dy}(y)$;

c) suppose that g is monotonic and that for some function h and all x in the range I of X we have $y = g(x)$ if and only if $x = h(y)$;

d) assume that h has first derivative $(dh/dy)(y)$. Then the PDF of Y in the region where $f_Y(y) > 0$ is given by $f_Y(y) = f_X(h(y)) \left|\frac{dh}{dy}(y)\right|$.

Problem 1. Let X be a random variable that takes values from 0 to 3 with equal probability $1/4$.

(a) Find the PMF of the random variable $Y = X^3 + 2$.

(b) Find expectation of Y.

Solution:

4	

Problem 2. Let X be the RV with PDF $f_X(x) = x^2$ in the interval $[0, 3]$ and $Y = 2X + 3$. Find $f_Y(y)$ and expected value of Y. Show graphically $f_X(x)$ and $f_Y(y)$.

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Solution:

Problem 3. Suppose X is uniformly distributed between 0 and 1. Find the PDF of $Y = e^X$. Show graphically all PDFs.

Solution:

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